Statistics and probability

Probability

Millions of Australians play lotto or buy lottery tickets in the hope of winning lots of money, but the chance of winning any of these games is very very small. Lotto is a national game of chance in which 6 balls are randomly selected from a barrel of balls numbered 1 to 45. To win, you must predict the correct 6 numbers to share in a first prize of at least one million dollars. However, the chance of doing this is 1 in 8 145 060, which means that even if you played 100 Lotto games per week, it would take you, on average, 1561 years to win it.
### Chapter outline

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### Wordbank

**at least** Referring to the smallest number, for example, 'at least 2' means 2, 3, 4, …, that is, '2 or more'.

**complementary event** The 'opposite' event (for example the complementary event to selecting an Ace from a deck of cards is not selecting an Ace).

**expected frequency** The expected number of times an event will occur over repeated trials.

**mutually exclusive events** Events or categories that have no items in common.

**trial** One go or run of a repeated probability experiment, for example, one roll of a die.

**two-way table** A table that shows the number of items belonging to overlapping categories.

**Venn diagram** A diagram that uses circles (usually overlapping) to group items into categories.
In this chapter you will:

- assign probabilities to the outcomes of events and determine probabilities for events
- identify complementary events and use the sum of probabilities to solve problems
- describe events using language of ‘at least,’ exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’
- represent events in two-way tables and Venn diagrams and solve related problems
- recognise the difference between mutually and non-mutually exclusive events
- convert representations of the relationship between attributes in Venn diagrams to two-way tables
- solve probability problems involving single-step experiments such as card, dice and other games
- compare observed frequencies across experiments with expected frequencies

SkillCheck

1. Convert each number into a decimal.
   a. \( \frac{17}{20} \)   b. \( \frac{3}{8} \)   c. 2%   d. 30%

2. List all the possible outcomes for each situation.
   a. tossing a coin   b. rolling a die
   c. the gender of a baby   d. the colour of traffic lights
   e. the result of a football game   f. the result of a driving test

3. Convert each number into a percentage.
   a. 0.7   b. 0.56   c. \( \frac{2}{3} \)   d. \( \frac{6}{25} \)

4. Evaluate each expression.
   a. \( 1 - \frac{1}{3} \)   b. \( 1 - \frac{2}{5} \)   c. \( 1 - \frac{3}{8} \)

5. Which term best describes the chance that the next baby born in Australia is a girl? Select the correct answer A, B, C, or D.
   A. certain   B. definite   C. even chance   D. probable

6. Convert each number into a simplified fraction.
   a. 0.28   b. 0.02   c. 64%   d. 80%

7. Is the chance of each event greater than or less than \( \frac{1}{2} \)?
   a. You being home by 4 p.m. this afternoon
   b. You winning Lotto one day
   c. You listening to the radio today
   d. You learning to drive next year
   e. You going interstate this year
   f. You sending a text message on your mobile phone today
In probability (chance) situations, the set of all possible outcomes is called the **sample space**. For example, if a coin is tossed, the sample space is {heads, tails}. If each outcome has an equal chance, then we say that each outcome is **equally likely**.

An **event** consists of one or more outcomes. For example, the event that a baby is born in a summer month consists of the outcomes {November, December, January}. An event has the symbol $E$, and we can calculate its probability as a fraction.

### Summary

$P(E)$ means ‘the probability of an event, $E$ (occurring)’. If all possible outcomes are **equally likely**, then:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

or

$$P(E) = \frac{\text{number of outcomes matching } E}{\text{number of outcomes in the sample space}}$$

A **favourable outcome** is one of the outcomes in the event that you want, whose probability you are calculating.

The **probability** of an event can be written as a fraction, decimal or percentage. For example, the probability of tossing tails on a coin can be written as $\frac{1}{2}$, 0.5 or 50%.

### Example 1

a. Write the sample space for this spinner.

b. Is each outcome equally likely?

c. Find the probability that the spinner lands on red.

d. Find the probability that the spinner lands on a ‘traffic light’ colour.

#### Solution

a. The sample space is {red, yellow, green, blue}.

b. Each coloured region is equal in size ($\frac{1}{4}$ of the circle), so each outcome is equally likely.

c. $P(\text{red}) = \frac{1}{4}$  

   **One chance in 4.**

d. $P(\text{traffic light colour}) = P(\text{red or yellow or green})$

   $$= \frac{3}{4}$$

   **3 favourable outcomes out of 4.**
The language of probability

<table>
<thead>
<tr>
<th>Probability term</th>
<th>In Example 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>An experiment is a situation involving chance that leads to results called outcomes.</td>
<td>Spinning a spinner</td>
</tr>
<tr>
<td>A trial is one go or run of the experiment.</td>
<td>One spin of the spinner</td>
</tr>
<tr>
<td>An outcome is the result of an experiment.</td>
<td>The spin landing on red</td>
</tr>
<tr>
<td>The sample space is the set of all possible outcomes.</td>
<td>{red, yellow, green, blue}</td>
</tr>
<tr>
<td>An event is one or more outcomes of an experiment.</td>
<td>The arrow landing on a ‘traffic light’ colour: red, yellow, or green</td>
</tr>
<tr>
<td>In a random experiment, every possible outcome has the same chance of occurring.</td>
<td>All spins on this spinner are random because every colour has the same chance</td>
</tr>
</tbody>
</table>

The range of probability

Because probability is a proper fraction, its value must range from 0 to 1, or as a percentage, from 0% to 100%.

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<table>
<thead>
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<tbody>
<tr>
<td>no way</td>
<td>not likely</td>
<td>even chance</td>
<td>almost definitely</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>must happen</td>
</tr>
<tr>
<td>impossible</td>
<td>$\frac{1}{2}$</td>
<td>certain</td>
<td></td>
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</tbody>
</table>

Example 2

A die is rolled. Find the probability that the number rolled is:

- a divisible by 3
- b a factor of 6, as a decimal
- c less than 7
- d at least 4, as a percentage

Solution

There are 6 possible outcomes when a die is rolled: {1, 2, 3, 4, 5, 6}

- a $P($divisible by 3$) = \frac{2}{6} = \frac{1}{3}$, 2 numbers (3 and 6).
- b $P($a factor of 6$) = \frac{4}{6} = \frac{2}{3} = 0.6$, 4 numbers (1, 2, 3, 6).
- c $P($less than 7$) = \frac{6}{6} = 1$, All outcomes are less than 7: a certain event.
- d $P($at least 4$) = \frac{3}{6} = \frac{1}{2} = 50\%$, 3 numbers (4, 5, 6).

‘at least 4’ means the smallest number is 4; that is, 4 or more.
Exercise 9-01  Probability

1 For each spinner:
   i write down the sample space
   ii for one spin, calculate the probability that the spinner lands on red.

   a
   ![Spinner a]

   b
   ![Spinner b]

   c
   ![Spinner c]

   d
   ![Spinner d]

   e
   ![Spinner e]

   f
   ![Spinner f]

2 For each experiment, count the number of possible outcomes and state whether each outcome
   is equally likely.
   a tossing a coin
   b the result of a rugby match when Australia plays New Zealand
   c the first letter of a person’s name
   d the gender of a baby
   e the last digit of a car number plate
   f the result of a driving test

3 List the outcomes in each event.
   a rolling an odd number on a die
   b selecting a vowel from the letters of the alphabet
   c having a house number greater than 4 but less than 10
   d having a birthday in a month beginning with A
   e living in a state of Australia
   f being in a high school grade

4 A money box contains four $2 coins, three $1 coins, two 50c coins, six 20c coins and five 10c
   coins. It is shaken and one coin falls out at random. Calculate each probability below.
   a \( P(50c \text{ coin}) \)
   b \( P(\$1 \text{ coin}) \), as a decimal
   c \( P(10c \text{ or } 20c \text{ coin}), \text{ as a percentage} \)
   d \( P(\text{not a } 10c \text{ coin}) \)
   e \( P(\text{gold coin}), \text{ as a decimal} \)
   f \( P(\text{a coin under } \$1), \text{ as a percentage} \)
5 A jar of lollies contains 5 red, 3 green, 6 yellow and 2 blue lollies. Taylor selects one lolly from the jar at random. Calculate each of the following probabilities.
   a $P$(green)
   b $P$(yellow or blue)
   c $P$(traffic light colour), as a decimal
   d $P$(not yellow), as a percentage
6 What is the probability that a person randomly chosen has a birthday in a month beginning with the letter J? Select the correct answer A, B, C or D.
   A 1/12
   B 1/6
   C 1/4
   D 1/3
7 NSW is playing Queensland in a rugby league match.
   a List the sample space for the outcomes of the match.
   b Are all outcomes equally likely? Explain your answer.
8 A packet of jelly beans has 4 yellow, 3 red, 6 green and 3 black jelly beans remaining. You tip the packet and one jelly bean rolls out at random. What is the probability that it is not a red jelly bean? Select the correct answer A, B, C or D.
   A 4/12
   B 3/16
   C 3/13
   D 13/16
9 John is holding 12 playing cards, as shown. His friend Lara picks a card without looking.
   a How many fives are held in the 12 cards?
   b List all the possible outcomes that Lara could pick.
   c Copy and complete this table.
<table>
<thead>
<tr>
<th>Outcome</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability as a fraction</td>
<td>1/4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability as a decimal</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability as a percentage</td>
<td>25%</td>
<td></td>
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</tbody>
</table>
10 Linda has two $5 notes, four $10 notes and three $20 notes in her wallet. If she selects one note at random, what is the probability that it is a $20 note? Select the correct answer A, B, C or D.
   A 3/35
   B 60/110
   C 1/3
   D 20/35
11 Match each probability value to its correct description.
   a 1/2
   b 0
   c 90%
   d 1
   e 3/4
   f 0.1
   g 0.6
   h 2%
   A cannot happen
   B better than average chance
   C even chance
   D good chance
   E very likely
   F almost impossible
   G slim chance
   H must happen
12 The 52 cards in a standard deck of playing cards are shown below, divided evenly into four suits: hearts, diamonds, clubs, spades.

Hearts: \[ \text{ } \]

Diamonds: \[ \text{ } \]

Clubs: \[ \text{ } \]

Spades: \[ \text{ } \]

The cards are shuffled and one is taken out at random.

a How many outcomes are in the sample space?

b Is each outcome equally likely?

c How many Aces are in the deck?

d How many hearts cards are there?

e How many red 7s are there?

f Find the probability of selecting:

i a red card

ii a clubs card

iii the King of spades

iv a Queen

v a card with an even number

vi a black picture card

13 The weather forecaster says that the probability of a rainy day in April is 30%.

a Is this a high probability or a low probability?

b About how many rainy days are expected in April?

14 Your maths teacher calls out a name randomly from your class roll. What is the probability that it is:

a your name?

b a girl’s name?

c someone aged 14?

d someone with brown hair?

15 An Esky contains 8 cans of lemonade, 5 cans of orange drink and 2 cans of lime drink. How many cans of cola must be added so that the probability of randomly selecting:

a a lemonade is 50%?

b an orange drink is 0.25?

c a lime drink is \( \frac{1}{12} \)?

d a cola can is \( \frac{2}{5} \)?

Investigation: Complementary events

In everyday life, we use the word ‘complementary’ to describe things that go together and ‘complete the picture’ when they are together. For example, when dressing for an occasion:

- a shirt and a matching tie complement each other
- a dress and a matching pair of shoes complement each other

Remember also that ‘complementary’ angles add to 90°.

In probability, complementary events are events that together make up all the possible outcomes.
Ancient dice

Objects similar to dice were used by people in prehistoric times to cast magic spells or predict the future. These were made of the anklebones of sheep, buffalo, or other animals. Dice have also been found in Egyptian tombs dating from 2000 BCE and Chinese excavations dating from 600 BCE. The ancient Greeks and Romans made dice similar to ours out of bone, ivory, stone and metal. Dice games were first studied by the Italian mathematician Girolamo Cardano in the 16th century, and so the study of probability was born.

There are two kinds of modern dice. **Perfect dice** have sharp edges and corners and are used mostly in gambling casinos, while round-cornered dice are generally used to play social and board games.

**Investigate the rules for placing the numbers on the faces of a die.**
9-02 Complementary events

In any situation, the probabilities of all possible outcomes must add to 1. **Complementary events** are events that together make up all the possible outcomes, such as when tossing a coin, a head and a tail. The complement of an event \( E \), are all those outcomes that are not \( E \), or that are the ‘opposite’ of \( E \).

Because an event and its complement covers all possible outcomes, the sum of their probabilities must equal 1.

**Summary**

\[
P(E) + P(\text{not } E) = 1 \\
\text{or } P(\text{not } E) = 1 - P(E) \\
\text{or } P(\text{complementary event}) = 1 - P(\text{event}) \\
\text{or } P(\text{event not occurring}) = 1 - P(\text{event occurring})
\]

If the probability is written in percentage form, then \( P(\text{not } E) = 100\% - P(E) \)

**Example 3**

A pack of 20 cards contains 10 red, 6 yellow and 4 green cards. One card is drawn from the pack at random. Find the probability that this card is:

- **a** yellow
- **b** not yellow
- **c** not green.

**Solution**

- **a** \( P(\text{yellow}) = \frac{6}{20} = \frac{3}{10} \)
- **b** \( P(\text{not yellow}) = 1 - P(\text{yellow}) \\
  = 1 - \frac{3}{10} \\
  = \frac{7}{10} \)

  Note that \( P(\text{yellow}) + P(\text{not yellow}) = \frac{3}{10} + \frac{7}{10} = 1 \), which covers all possible outcomes.

- **c** \( P(\text{green}) = \frac{4}{20} = \frac{1}{5} \)

  \( P(\text{not green}) = 1 - P(\text{green}) \\
  = 1 - \frac{1}{5} \\
  = \frac{4}{5} \)

  Note that \( P(\text{green}) + P(\text{not green}) = \frac{1}{5} + \frac{4}{5} = 1 \), which covers all possible outcomes.
Exercise 9-02 Complementary events

1. Write the complement of each event below.
   a. Tossing tails on a coin
   b. Cloudy day tomorrow
   c. Selecting a white chocolate from a box containing white and brown chocolates
   d. Rolling a 6 on a die
   e. Winning a game of hockey
   f. Selecting a hearts card from a deck of cards
   g. Being born in winter
   h. Being under 15 years old

2. A die is rolled. What is the probability that the result is:
   a. a 3?
   b. not a 3?
   c. not even?
   d. not prime?
   e. at least 2?
   f. at most 4?

3. On the shelf there are 6 books: 3 Mathematics, 2 History and 1 Sport. If one book is chosen at random, what is the probability of selecting:
   a. a History book?
   b. a book that is not History?
   c. a book that is not Sport?
   d. a book that is not Science?

4. A car park has 450 cars, 100 motor bikes and 10 buses. One of them is selected at random.
   a. What is the probability that it is a car?
   b. What is the probability that it is not a car? Select the correct answer A, B, C or D.
      A: \( \frac{45}{56} \)
      B: \( \frac{110}{450} \)
      C: \( \frac{10}{560} \)
      D: \( \frac{11}{56} \)

5. What is the probability that your maths teacher was born in a month:
   a. beginning with the letter J?
   b. that does not begin with the letter J?

6. What is the decimal probability that a mobile phone number selected at random does not end in 0 or 1?

7. Tahnee buys 5 tickets in a raffle in which 1000 tickets are sold and there is only one prize. What is the probability of Tahnee not winning the prize?

8. A jar contains 40 red, 25 blue, 50 black and 15 white jelly beans. One jelly bean is selected at random. What is the probability that it is:
   a. white?
   b. not white?
   c. not yellow?
   d. not blue or black?
   e. not red?
   f. not black or white?

9. Write the probability of the event that is complementary to each of the following events.
   a. The probability of choosing a Jack from a pack of cards is \( \frac{1}{13} \).
   b. The chance of shooting a basketball hoop is 61%.
   c. The probability of winning a prize is 0.15.
10. In a bag of toy cars there are only three colours: red, blue and white. If you take out a car at random, the chance of it being red is 0.4, and the chance of it being white is 0.25.
   a. What is the chance of selecting ‘red or white’?
   b. What is the chance of the car you select being blue?
   c. If the bag holds 40 cars, how many of each colour would you expect to find?
   d. What is the chance of the car you select being pink?

11. Four students, Sue, Liam, Emily and Matt, write their names on cards and place the cards in a bag. A card is chosen, without looking, to select the class captain.
   a. Find the probability that Emily was not chosen.
   b. Find the probability that the captain is a boy.
   c. What is the chance that the captain is not a boy?
   d. What is the chance that the captain is the teacher?

12. The probability that it will rain this weekend is 85%. What is the probability that it won’t rain?

13. Which of the following is the complementary event to ‘winning a race’? Select the correct answer A, B, C or D.
   A. coming last
   B. coming second or third
   C. not winning the race
   D. coming second

14. A game involves throwing a coin into a grid of squares. For a player to win, the coin must land in a red square. If the coin lands outside the grid or between squares, the throw is not counted and the player has another try.

   For each throw, what is the probability of:
   a. not winning?
   b. winning?

15. The letters of the word PROBABILITY are written on separate cards and one is selected randomly. What is the probability that a letter drawn out is:
   a. not P?
   b. not a vowel?
   c. not I?
   d. not A or B?

16. In a family there are Mum and Dad, two daughters and a son. Each day, they take it in turns to wash the dishes after dinner. If you visit the family, what is the chance that the person washing up is:
   a. male?
   b. not a parent?
   c. not Mum?
   d. not male?
A **Venn diagram** is a diagram that uses circles (usually overlapping) to group items into categories. A rectangle represents the whole group while the circles represent categories. Items common to two or more categories are placed in the intersection (overlapping region) of the circles. The Venn diagram was invented in 1880 by English mathematician and priest, John Venn (1834–1923).

**Example 4**

A group of people were surveyed on the type of vehicle they drove, and the results are shown on the Venn diagram below.

- How many people were surveyed?
- How many people drive a car?
- How many people drive a car or ride a motorbike?
- Who drives a car and rides a motorbike?
- Who only rides a motorbike?
- Why is Sid outside of the circles?

**Solution**

- 12 people were surveyed.
- 7 people drive a car.
- 11 people drive a car or ride a motorbike.
- Sue and Bill can drive a car and ride a bike.
- Cathy, Erica, John and Ron can only ride a bike.
- Sid does not drive a car or ride a bike.

**Mutually exclusive vs overlapping events**

Sometimes, two categories must be represented on a Venn diagram as two separate circles because it is not possible for them to overlap. Here is an example:

In this case, it is not possible to be male and female. This is an example of **mutually exclusive events**. Mutual means ‘shared feature’ and exclusive means ‘belonging to one group only’ (such as an exclusive party or an exclusive news story). However, most Venn diagrams (as in Examples 4 and 5) describe **overlapping events**, or **non-mutually exclusive events**.
‘And’ vs ‘or’

For two categories or events A and B, the phrase ‘A and B’ means to have both of them occurring together. For example, ‘to drive a car’ and ‘to ride a motorbike’ in Example 4 means to do both things.

If A and B are overlapping, the phrase ‘A or B’ means to have A or B or both. For example, ‘to drive a car’ or ‘to ride a motorbike’ in Example 4 means to drive a car only, or to ride a motorbike only, or to do both. In this case, ‘A or B’ actually includes ‘A and B’ so this is an example of an inclusive ‘or’.

If A and B are mutually exclusive, the phrase ‘A or B’ means to have A only or B only (but not both). For example, ‘male’ or ‘female’ means to be male, or female, but not both. In this case, ‘A or B’ excludes ‘A and B’ so this is an example of an exclusive ‘or’.

Example 5

30 students in a class were surveyed on how they relaxed after school. Here are the results:

- 18 play on their computer
- 15 play sport
- 6 play sports and play on their computer
  a Show this information on a Venn diagram.
  b If one student is selected at random from this class, find the probability that the student:
    i plays sport but not on their computer
    ii plays sport or on their computer but not both

Solution

a When drawing Venn diagrams always begin with the intersection.
- 6 students belong to both groups
- 18 play on the computer but 6 have already been counted, so 18 – 6 = 12 play on the computer only
- 15 play sport but 6 have already been counted, so 15 – 6 = 9 play sport only
- 12 + 6 + 9 = 27 but there are 30 students in the class, which means that 3 students do not belong to either group

b i \( P(\text{plays sport but not computer}) = \frac{9}{30} = \frac{3}{10} \)

ii \( P(\text{plays sport or computer but not both}) = \frac{12 + 9}{30} = \frac{21}{30} = \frac{7}{10} \)
Exercise 9-03 Venn diagrams

A group of students were asked whether they liked Maths or Science at school. The Venn diagram shows the results of this survey.

![Venn diagram showing Maths and Science]

How many students:
- **a** were surveyed?
- **b** liked Maths but not Science?
- **c** liked Maths or Science?
- **d** liked Maths or Science but not both?
- **e** did not like either subject?
- **f** liked Maths and Science?

The Venn diagram below shows the sports played by a number of female students.

![Venn diagram showing Netball and Hockey]

- **a** Are netball and hockey mutually exclusive or not?
- **b** How many students play neither netball nor hockey?
- **c** How many students play netball or hockey?
- **d** Find the total number of students surveyed.
- **e** How many students play netball and hockey?

This Venn diagram compares the categories Female and Left-handed.

![Venn diagram showing Female and Left-handed]

- **a** Is Female and Left-handed mutually exclusive or not?
- **b** Find the total number of people represented in the diagram.
- **c** How many left-handed females are there?
- **d** What is the decimal probability (correct to 3 decimal places) that a person randomly chosen from this group is:
  - **i** female?
  - **ii** female but not left-handed?
  - **iii** female or left-handed?
  - **iv** a left-handed male?
- **e** Describe the type of person that would be represented outside the two circles on the Venn diagram.

See Example 4

See Example 5
4. A survey was carried out by an ice-cream shop to decide whether chocolate or strawberry was the more popular flavour.

![Ice cream image]

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Strawberry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>57</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>248</td>
<td>15</td>
</tr>
</tbody>
</table>

a. How many people were surveyed?

b. How many people liked strawberry or chocolate but not both?

c. How many people liked neither strawberry nor chocolate?

d. If a person is randomly chosen from the survey, what is the probability that the person likes both chocolate and strawberry? Select the correct answer A, B, C or D.

A \[ \frac{57}{440} \]  
B \[ \frac{368}{440} \]  
C \[ \frac{425}{440} \]  
D \[ \frac{57}{425} \]

5. The Tourism Council surveyed 130 people to find whether they preferred New South Wales or Queensland as a holiday destination.

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<tbody>
<tr>
<td>NSW</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>Queensland</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>NSW and Queensland</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

a. Construct a Venn diagram to represent the results.

b. How many people prefer NSW or Queensland?

c. How many people prefer NSW or Queensland but not both?

d. What is the probability that a person prefers NSW exclusively?

e. What is the probability that a person does not prefer NSW or Queensland?

6. a. Draw a Venn diagram representing the eye colours of this group of students.

<table>
<thead>
<tr>
<th>Eye Colour</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue eyes</td>
<td>32</td>
</tr>
<tr>
<td>Brown eyes</td>
<td>38</td>
</tr>
<tr>
<td>Neither blue nor brown eyes</td>
<td>10</td>
</tr>
</tbody>
</table>

b. Are these groups mutually exclusive or not? Why?

c. How many students are in the group?

d. How many students have blue or brown eyes?

e. How many students have blue and brown eyes?

f. What is the percentage probability that a student chosen at random from this group does not have brown eyes?
A shopkeeper surveyed the first 30 customers to see what they bought.
20 bought milk
13 bought bread
1 bought neither milk nor bread

a Show this information on a Venn diagram.
b How many customers bought milk and bread?
c How many customers bought milk only?
d What is the probability that a customer randomly chosen:
   i bought bread only?
   ii did not buy milk?
   iii bought milk or bread?
   iii bought milk or bread but not both?

A two-way table is another way of grouping items into overlapping categories, especially when there are many overlaps that cannot be represented by Venn diagrams easily.

Example 6
Fifty students were surveyed on whether they liked dogs or cats more as pets. The results were sorted into a two-way table.
a How many students do not like dogs or cats?
b How many students like cats?
c How many boys like cats?
d How many students:
   i like cats or are boys?
   ii like cats or are boys but not both?
e What is the probability that a student selected at random from the survey likes dogs?

Solution
a \[ 12 + 4 + 11 + 15 = 42 \]
Students who do not like dogs or cats = \( 50 - 42 \)
\[ = 8 \]
50 students in survey.
b Students who like cats = \( 4 + 15 \)
\[ = 19 \]
c Boys who like cats = 4
d i Students who like cats or who are boys = \( 12 + 4 + 15 \)
\[ = 31 \]
ii Students who like cats or who are boys but not both = \( 12 + 15 \)
\[ = 27 \]
e  Students who like dogs = 12 + 11
   = 23

   Probability of selecting a student who likes dogs = \( \frac{23}{50} \)

Example 7

A group of 50 Year 9 students were grouped in a Venn diagram according to whether they took Chinese or Art as an elective subject.

Represent this information on the two-way table below.

<table>
<thead>
<tr>
<th></th>
<th>Chinese</th>
<th>Not Chinese</th>
</tr>
</thead>
<tbody>
<tr>
<td>Art</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Not Art</td>
<td>21</td>
<td>8</td>
</tr>
</tbody>
</table>

Solution

From the Venn diagram:

- 9 students take Chinese and Art
- 21 students take Chinese but not Art
- 12 students take Art but not Chinese
- 8 students do not take Chinese or Art

<table>
<thead>
<tr>
<th>Chinese</th>
<th>Not Chinese</th>
</tr>
</thead>
<tbody>
<tr>
<td>Art</td>
<td>9</td>
</tr>
<tr>
<td>Not Art</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>
Exercise 9-04  Two-way tables

See Example 6

1 A primary school class was surveyed on whether its students could swim. The results are shown below.

<table>
<thead>
<tr>
<th></th>
<th>Can swim</th>
<th>Cannot swim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>13</td>
<td>2</td>
</tr>
<tr>
<td>Girls</td>
<td>93</td>
<td>3</td>
</tr>
</tbody>
</table>

a How many students are in the class?
b How many students are boys or cannot swim?
c How many students are boys and cannot swim?
d What is the probability that a student randomly selected from this class is a girl?
e What is the probability that a student selected at random is:
   i a non-swimmer?  ii a girl who can swim?

2 The players of a soccer club were divided into groups according to their age and weight.

<table>
<thead>
<tr>
<th></th>
<th>Heavy</th>
<th>Light</th>
</tr>
</thead>
<tbody>
<tr>
<td>Junior</td>
<td>64</td>
<td>96</td>
</tr>
<tr>
<td>Senior</td>
<td>144</td>
<td>32</td>
</tr>
</tbody>
</table>

a How many players does the club have?
b How many players are juniors or light?
c How many players are juniors or light but not both?
d What is the probability that a player selected at random is:
   i a senior?  ii a junior and heavy?  iii is a senior or light?

3 This incomplete table describes the audience watching a movie at a cinema.

<table>
<thead>
<tr>
<th></th>
<th>Under 18</th>
<th>Over 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td></td>
<td>142</td>
</tr>
<tr>
<td>Male</td>
<td>45</td>
<td></td>
</tr>
</tbody>
</table>

a Copy and complete the table.
b How many males were in the audience?
c How many under 18 females were there?
d If a person is selected at random from the audience, what is the probability that the person:
   i is male and over 18?  ii is male or over 18?
   iii is male or over 18 but not both?  iv is over 18?
4 A group of children were asked whether they liked carrots. The table shows the results.

<table>
<thead>
<tr>
<th>Likes carrots</th>
<th>Dislikes carrots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>75</td>
</tr>
<tr>
<td>Girls</td>
<td>40</td>
</tr>
</tbody>
</table>

a Copy and complete the table.

b How many children dislike carrots?

c What is the probability that a child randomly selected is:
   i a girl?
   ii a boy or dislikes carrots?
   iii a boy and dislikes carrots?

d What is the chance that a child randomly selected is a girl and likes carrots? Select the correct answer A, B, C or D.

A \( \frac{29}{62} \)  
B \( \frac{11}{31} \)  
C 1  
D \( \frac{7}{31} \)

5 The Venn diagram below shows the number of students who participate in swimming or bowling regularly. Copy and complete the two-way table for this data.

6 The Venn diagram compares two features of the cars on display at Rusty Motors.

a Copy and complete the two-way table.

b A car is chosen at random from the car yard. What is the probability that it has:
   i GPS navigation?
   ii 4 doors?
   iii 4 doors and GPS navigation?
   iv no GPS navigation?
A group of university students were tested to see if they needed glasses. The table shows the results.

<table>
<thead>
<tr>
<th></th>
<th>Needs glasses</th>
<th>Does not need glasses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>22</td>
<td>82</td>
</tr>
<tr>
<td>Female</td>
<td>85</td>
<td>98</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>145</strong></td>
<td></td>
</tr>
</tbody>
</table>

a  Copy and complete the table.

b  If a student is chosen randomly from this group, what is the probability that the student:

i  is female?

ii is female and needs glasses?

iii is female or needs glasses?

iv is male or does not need glasses?

v is male or does not need glasses but not both?

Technology  Rolling a die

In this activity, a graphics calculator is used to simulate the rolling of a die.

Note: This activity uses a Casio graphics calculator.

1 Copy the table shown below, for the possible outcomes.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Number of times rolled</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

2 Using the RUN mode, enter the following formula: \( \text{Int}(\text{Ran#} \times 6 + 1) \) as shown below.

3 Repeat the simulation 20 times and record the results in your table.

4 Are certain numbers more likely to be rolled than others? (For example is a 2 more likely to be rolled than a 5?) Do your results reflect this?

5 Compare your results with the simulated results of other members in your class. Are they similar or different? Are they what you and your classmates expected? Discuss.
Mental skills 9  Maths without calculators

Multiplying or dividing by a multiple of 10

1 Study each example.
   a  \(4 \times 700 = 4 \times 7 \times 100 = 28 \times 100 = 2800\)
   b  \(5 \times 60 = 5 \times 6 \times 10 = 30 \times 10 = 300\)
   c  \(12 \times 40 = 12 \times 4 \times 10 = 48 \times 10 = 480\)
   d  \(3.2 \times 30 = 3.2 \times 3 \times 10 = 9.6 \times 10 = 96\)  (by estimation, \(3 \times 30 = 90 \approx 96\))
   e  \(4.5 \times 50 = 4.5 \times 5 \times 10 = 22.5 \times 10 = 225\)  (by estimation, \(5 \times 50 = 250 \approx 225\))
   f  \(9.4 \times 200 = 9.4 \times 2 \times 100 = 18.8 \times 100 = 1880\)  (by estimation, \(9 \times 200 = 1800 \approx 1880\))

2 Now evaluate each product.
   a  \(8 \times 2000\)
   b  \(3 \times 70\)
   c  \(11 \times 900\)
   d  \(2 \times 300\)
   e  \(4 \times 4000\)
   f  \(5 \times 80\)
   g  \(7 \times 70\)
   h  \(1.3 \times 40\)
   i  \(2.5 \times 600\)
   j  \(6.8 \times 200\)
   k  \(3.9 \times 50\)
   l  \(4.4 \times 4000\)

3 Study each example.
   a  \(8000 \div 400 = 8000 \div 100 \div 4 = 80 \div 4 = 20\)
   b  \(200 \div 50 = 200 \div 10 \div 5 = 20 \div 5 = 4\)
   c  \(6000 \div 20 = 6000 \div 10 \div 2 = 600 \div 2 = 300\)
   d  \(282 \div 30 = 282 \div 10 \div 3 = 28.2 \div 3 = 9.4\)
   e  \(3520 \div 40 = 3520 \div 10 \div 4 = 352 \div 4 = 88\)
   f  \(8940 \div 200 = 8940 \div 100 \div 2 = 89.4 \div 2 = 44.7\)

4 Now evaluate each quotient.
   a  \(560 \div 70\)
   b  \(2500 \div 50\)
   c  \(3200 \div 400\)
   d  \(440 \div 20\)
   e  \(160 \div 40\)
   f  \(1500 \div 30\)
   g  \(450 \div 50\)
   h  \(744 \div 80\)
   i  \(2550 \div 300\)
   j  \(846 \div 200\)
   k  \(576 \div 60\)
   l  \(2040 \div 50\)

9-05 Probability problems

The following exercise involves all the probability ideas that have been covered so far.

Exercise 9-05 Probability problems

1 There were 28 students at the SRC conference: 17 were born in Australia, 3 in Italy, 4 in Vietnam, 1 in Japan and 3 in Sweden. One student was chosen at random. Find the probability that the student was:
   a  born in Australia
   b  born in Europe
   c  not born in Japan.
2 The letters of the word SUCCESS are written on cards. A card is selected at random and the letter noted.
   a List the sample space.     b Is each letter equally likely to be selected? Explain.
   c Find the chance that the letter chosen:
      i is an S     ii is a vowel     iii is not a C     iv is also a letter of the word FAIL.
3 Alex selects one sock at random from a bag containing two black, two blue and two red socks.
   a List the sample space.
   b Find the probability that Alex selects:
      i a blue sock     ii a black sock     iii a pink sock
   c What is the complementary event to choosing a red sock? What is the probability of this event?
4 A spinner is evenly divided into 5 sections numbered 1, 2, 3, 4 and 5. For one spin, find the probability, as a percentage, that it lands on:
   i 2     ii an even number     iii a number less than 5     iv a number at most 5
5 Sophia bought four tickets in a lottery in which there were 100 000 tickets. What is the probability that she won first prize? Select the correct answer A, B, C or D.
   A 0.000 0025     B 0.000 25     C 0.04     D 0.000 04
6 One ball is selected at random from a barrel of balls numbered 1 to 100. Find the probability that the number shown on the ball is:
   a 12     b greater than 40     c even     d at most 85
7 Stathis flipped a coin 7 times and a tail showed each time. What is the chance of a tail showing on the next toss?
8 A computer selected a random number from 1 to 15 inclusive. Find the probability that the number is:
   a odd     b not prime     c a multiple of 3 or 5     d a factor of 12
9 In a tennis tournament there are 60 players. Of these, 34 are from NSW, 20 are from Victoria and 6 are from Queensland. Find the probability that a player selected at random from the tournament:
   a is from NSW     b is from Queensland     c is not from Victoria     d is from Queensland or Victoria.
10 40 students at a school camp can select kayaking or bushwalking as an activity, or both. A total of 35 students chose kayaking, 20 chose bushwalking, including some who chose both. There weren’t any students who did not choose an activity.
   a Copy and complete the Venn diagram representing these students.
   b What is the probability that a student, chosen at random:
      i chose both activities?     ii did not choose bushwalking?
11 The probability that a carton of eggs contains any broken eggs is 0.1. Find the probability that a carton contains no broken eggs.

12 A sample of students from a sports high school were surveyed on whether they participated in hockey or judo. The results are shown in this two-way table.

<table>
<thead>
<tr>
<th>Judo</th>
<th>Not Judo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hockey</td>
<td>128</td>
</tr>
<tr>
<td>Not hockey</td>
<td>100</td>
</tr>
</tbody>
</table>

a Copy and complete the table.

b What is the probability that a student chosen randomly:

i does judo?  
iii does judo but not hockey?  
v does hockey or judo? 

ii doesn’t do hockey?  
iv does hockey and judo?  
vi does hockey or judo but not both?

### 9-06 Experimental probability

A probability calculated using the formula:

\[ P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \]

is more specifically called the **theoretical probability** (or **calculated probability**). We can also determine the probability of an event based on the results of an experiment or trial that has been repeated many times, such as the safety testing of different cars, or rely on past statistics, such as the number of rainy days in April. This type of probability is called **experimental probability** (or **relative frequency**).

**Summary**

**Experimental probability** has these formulas:

\[ P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}} \]

or \[ P(E) = \frac{\text{frequency of } E}{\text{total frequency}} \]

**Expected frequency** is the expected number of times an event will occur over repeated trials.

Expected frequency = theoretical probability \( \times \) number of trials.
Example 8

Declan rolled a die 60 times and recorded the results in a table.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
</tr>
</tbody>
</table>

a What is the theoretical probability of rolling a 5 or 6 on a die?
b For 60 rolls, what is the expected number of times of getting a 5 or 6? How does this compare with the actual number of times?
c What is the experimental probability of rolling a 5 or 6?

Solution

a \( P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3} \)

b Expected number of 5s or 6s = \( \frac{1}{3} \times 60 = 20 \)

From the table, the observed number of 5s and 6s = 8 + 11 = 19, which is close to the expected number, 20.

c Experimental \( P(5 \text{ or } 6) = \frac{8 + 11}{60} = \frac{19}{60} \)

Exercise 9-06 Experimental probability

1 Erica rolled a biased die 60 times and the number 2 came up 15 times.
   a What is the experimental probability of rolling a 2?
   b If the same die was rolled 500 times, what is the expected frequency of rolling a 2?

2 a Copy this table.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tail</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b Flip a coin 50 times and record the results in the table.

c Find, as a decimal, the experimental probability of flipping:
   i a head
   ii a tail

d Find, as a decimal, the theoretical probability of flipping:
   i a head
   ii a tail

e How do the experimental probabilities compare with the theoretical probabilities?
3 Josie spun this spinner 80 times and found the following results:

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Red</th>
<th>Green</th>
<th>Yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>44</td>
<td>25</td>
<td>11</td>
</tr>
</tbody>
</table>

a What is the theoretical probability of spinning red?
b For 80 spins, what is the expected number of times of spinning red? How does this compare with the actual number of times?
c What is the experimental probability of spinning red?
d What is the theoretical probability of spinning yellow?
e Find the expected frequency of spinning yellow over 80 spins and compare this with the observed frequency.
f What is the experimental probability of spinning yellow?

4 The matches in a sample of matchboxes were counted. The number of matches in each box are recorded below.

<table>
<thead>
<tr>
<th>Number of matches</th>
<th>48</th>
<th>49</th>
<th>50</th>
<th>51</th>
<th>52</th>
<th>53</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of matchboxes</td>
<td>3</td>
<td>6</td>
<td>20</td>
<td>16</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

a How many matchboxes were tested?
b What is the experimental probability of finding 51 matches in a box?
c What is the experimental probability of finding fewer than 50 matches in a box?
d In 1000 matchboxes, how many matchboxes would you expect to contain:
   i exactly 50 matches?    ii at least 50 matches?

5 a Copy this table.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b Roll a die 72 times and record the results in the table.
c Find the theoretical probability of rolling:
   i 4    ii a number greater than 2    iii an even number
d When rolling a die 72 times, what is the expected frequency of rolling:
   i 4?    ii a number greater than 2?    iii an even number?
e How do your expected frequencies compare with the actual frequencies from the table?
f Find the experimental probability of rolling:
   i 4    ii a number greater than 2    iii an even number
g If a die was rolled 1000 times, how many times should 4 come up:
   i based on the theoretical probability?
   ii based on the experimental probability?
6 Tamara tossed a coin many times and got 135 heads and 115 tails. Calculate, as a percentage, the experimental probability of tails with this coin.

7 A die was rolled 80 times, with the results shown below.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>11</td>
<td>13</td>
<td>9</td>
<td>13</td>
<td>12</td>
<td>22</td>
</tr>
</tbody>
</table>

a Is each outcome equally likely?
b Do you think this die is biased (unfair)? Give a reason for your answer.
c Write, as a percentage, the experimental probability of rolling a 1 on this die.
d If this die was rolled 100 times, how many times would you expect 3 to come up?

8 A pair of dice was rolled 50 times and their sum calculated each time. The results are shown in this table.

<table>
<thead>
<tr>
<th>Sum</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>9</td>
<td>6</td>
<td>8</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

a Find, as a decimal, the experimental probability of rolling a sum:
   i of 11
   ii of 5 or 6
   iii that is odd
   iv 6 or more
   v at most 6
   vi that is a composite number

b Which sum(s):
   i was most likely?
   ii had a probability of \(\frac{3}{25}\)
   iii was least likely?
   iv had a probability of 16%?
   v was second-most likely?
   vi had a probability of \(\frac{1}{10}\)?

---

**Technology Tossing a coin**

In this activity, a spreadsheet is used to simulate the tossing of a coin. The computer can quickly generate either a 1 (representing heads) or a 2 (tails). We will use the command RAND to generate these random numbers.

1 In cell A1 enter the label ‘Tossing a coin’.
2 In cell A2, enter the formula =INT(RAND()*2+1). Press Enter and either 1 or 2 should appear randomly in the cell.
3 Select cell A2 and Fill Down to cell A11 to generate random numbers (1 or 2), to simulate the tossing of a coin 10 times.
4 Your results will probably be different from those of other students in the class. Compare.
5 Copy the table below and in the first blank row record your results for Trials 1–10 (the numbers of heads and tails in your first 10 ‘tosses’).

<table>
<thead>
<tr>
<th>Trial</th>
<th>Number of heads</th>
<th>Number of tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11–20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21–30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31–40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>41–50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>51–60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>61–70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>71–80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>81–90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>91–100</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6 Simulate another 10 tosses of the coin by making the spreadsheet generate another set of random numbers. Do this by placing the cursor between the tops of columns A and B (so that it turns into a ‘double arrow’) and clicking.

7 Enter the results for Trials 11–20 in your table.

8 Repeat 8 more times so that you have 100 tosses of the coin recorded in the table.

9 a For 100 tosses of a coin, what is the expected frequency of heads based on the theoretical probability?

b Compare this with the actual frequency.

10 Calculate, as a decimal, the theoretical and experimental probabilities of tossing heads.

11 Compare your results with those of other students in your class. Briefly explain any differences and why they may have occurred.

12 Combine the results of students in your class to calculate the experimental probability of tossing heads, as a decimal. How does this compare with the theoretical probability?
A market research survey of 50 people shows that 42 people own a car, 20 own a digital tablet and 8 own a 3D TV. Of these:

- 4 own a car and a 3D TV
- 3 own a 3D TV and a tablet
- 14 own a car and a tablet
- 1 person has a car, a tablet and a 3D TV

Copy and complete the following Venn diagram to represent the survey results.

Two dice are rolled and the numbers are multiplied together to arrive at a score.

a Copy and complete this table to show all possible products.

<table>
<thead>
<tr>
<th>First die</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b How many different products are possible?

c Why isn’t each product equally likely?

d Which product is most likely?

e Which product is least likely?

f What is the probability of a product of:

i 6?  

ii 20?  

iii at least 20?

g Which product has a probability of $\frac{1}{12}$?

Three friends decide to play a game with two dice. Danielle wins if the sum of the numbers is 3 or 5, and Vanessa wins if the total is 6 or 8. Any other total means that Karla wins. Is the game fair? Explain your answer.

A committee of 2 people is to be selected from two boys (Paul and Sumeet) and two girls (Tash and Nadine).

a List all the committees you can form.

b If you were to choose a committee at random, what is the probability that it would include Tash?
### Language of maths

- at least
- certain
- complementary event
- exclusive
- expected frequency
- experimental probability
- improbable
- impossible
- inclusive
- likely
- mutually exclusive
- observed frequency
- outcome
- overlapping
- probable
- random
- relative frequency
- sample space
- theoretical probability
- trial
- two-way table
- unlikely
- Venn diagram

1. What is the meaning of **sample space**?
2. What is the **complementary event** to winning a soccer match?
3. What does it mean when a person is selected ‘at random’ for a survey?
4. Draw a Venn diagram with categories ‘Year 7 students’ and ‘Year 8 students’. Are these categories **mutually exclusive** or not?
5. What term means the number of times an event should occur over repeated trials?
6. Explain what ‘is right-handed or drives a car’ means exactly, given that they are **overlapping** events.

### Topic overview

- Write in your own words what you have learnt in this chapter.
- Write which parts of the chapter were new to you.
- Copy and complete:
  - The things I understand about probability are…
  - The things I am still not confident in doing in this chapter are…

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.
1. A die is rolled. Find the probability that the number that comes up is:
   a 1  b more than 2  c odd  d composite

2. a Write the sample space for this spinner.
   b Find, as a percentage, the probability that the number spun is:
      i 5  ii at least 5  iii 5 or less  iv less than 5

3. Write the complement of each event and its probability.
   a Choosing an Ace from a standard deck of cards
   b Rolling a factor of 6 on a die
   c Buying the winning ticket out of 1000 tickets sold

4. A golfer has a probability of 74% of sinking a putt. What is the probability that he will miss a putt?

5. A truck carries 240 boxes of lemonade, 305 boxes of soap, 335 boxes of paper and 120 boxes of pens. The driver chooses one box at random. What is the probability that it is:
   a a box of lemonade?
   b a box of pens or soap?
   c a box of paper?
   d not a box of paper?

6. The medical history of a group of children is shown in the following Venn diagram.
   a How many children are in the group?
   b How many children have had the mumps and the measles?
   c How many children have had the mumps or the measles but not both?
   d What is the probability that one child selected at random from this group has had:
      i the measles?
      ii the mumps or the measles?
      iii neither the mumps nor the measles?

7. A group of people were surveyed on whether they watched cricket or soccer on TV.
   The results were sorted into a two-way table.
   a How many people were surveyed?
   b How many people watch soccer?
   c How many people watch soccer or cricket?
   d What is the probability that a person selected at random from the survey:
      i watches neither soccer nor cricket?
      ii watches soccer or cricket but not both?
      iii does not watch cricket?
      iv does not watch soccer?
8 Represent the data in the Venn diagram on the two-way table. See Exercise 9-04

<table>
<thead>
<tr>
<th></th>
<th>Tall</th>
<th>Not tall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blond</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Not blond</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

9 An eight-sided die has 2 red, 3 white, 1 blue and 2 yellow faces. If the die is rolled, find the decimal probability that the face that comes up is:
   a blue   b yellow   c not red   d white or yellow

10 A die was rolled 80 times and the numbers 1 or 6 came up 25 times.
   a What is the experimental probability of rolling 1 or 6, as a percentage?
   b What is the theoretical probability of rolling 1 or 6, as a percentage?
   c Calculate the expected frequency of rolling 1 or 6 over 80 trials.

See Exercise 9-04
See Exercise 9-05
See Exercise 9-06